Key

AIM: How do we calculate the mean and standard deviation of the sum or difference of ***normal*** independent random variables? **(4-16D)** PRACTICE

Do Now:

1. Mr. Voss and Mr. Cull bowl every Tuesday night. Over the past few years, Mr. Voss's scores have been approximately Normally distributed with a mean of 212 and a standard deviation of 31. During the same period, Mr. Cull's scores have also been approximately Normally distributed with a mean of 230 and a standard deviation of 40. Assuming their scores are independent, what is the probability that Mr. Voss scores higher than Mr. Cull on a randomly-selected Tuesday night?

$V: N(212, 31)$

$C: N(230, 40)$

$D = V - C$

want $D \oplus$

$E(D) = E(V) - E(C)$

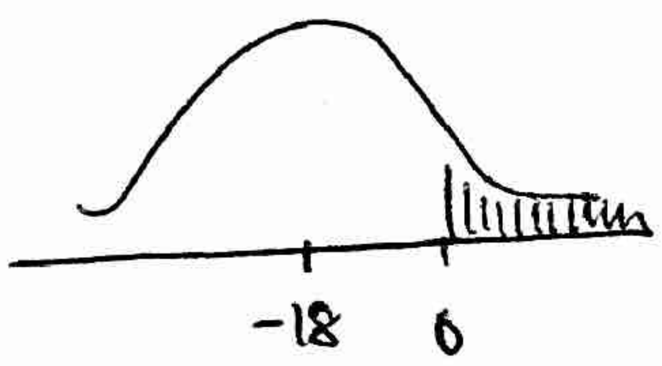
$= 212 - 230$

$= -18$

$SD(D) = \sqrt{Var(V-C)} = \sqrt{Var(V) + Var(C)}$

$= \sqrt{31^2 + 40^2} = \sqrt{2561} \approx 50.61$

$N(-18, 50.61)$



$z = \dfrac{0 - (-18)}{50.16}$

$z = 0.3557$

$\Rightarrow 0.3610 \approx 0.36$

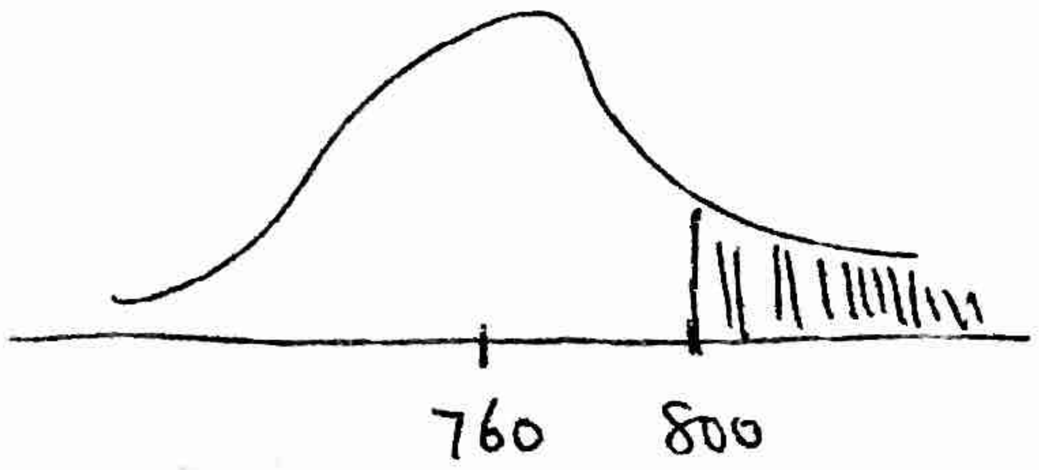
Practice (Group Work)

2. A summer resort rents rowboats to customers but does not allow more than four people to a boat . Each boat is designed to hold no more than 800 pounds . Suppose the distribution of adult males who rent boats, including their clothes and gear, is normal with a mean of 190 pounds and standard deviation of 10 pounds . If the weights of individual passengers are independent, what is the probability that a group of four adult male passengers will exceed the acceptable weight limit of 800 pounds?

$E(m_1 + m_2 + m_3 + m_4) = 4(190) = 760$

$SD(m_1 + m_2 + m_3 + m_4) = \sqrt{10^2(4)} = 20$

$N(760, 20)$



$z = \dfrac{800 - 760}{20} = 2$

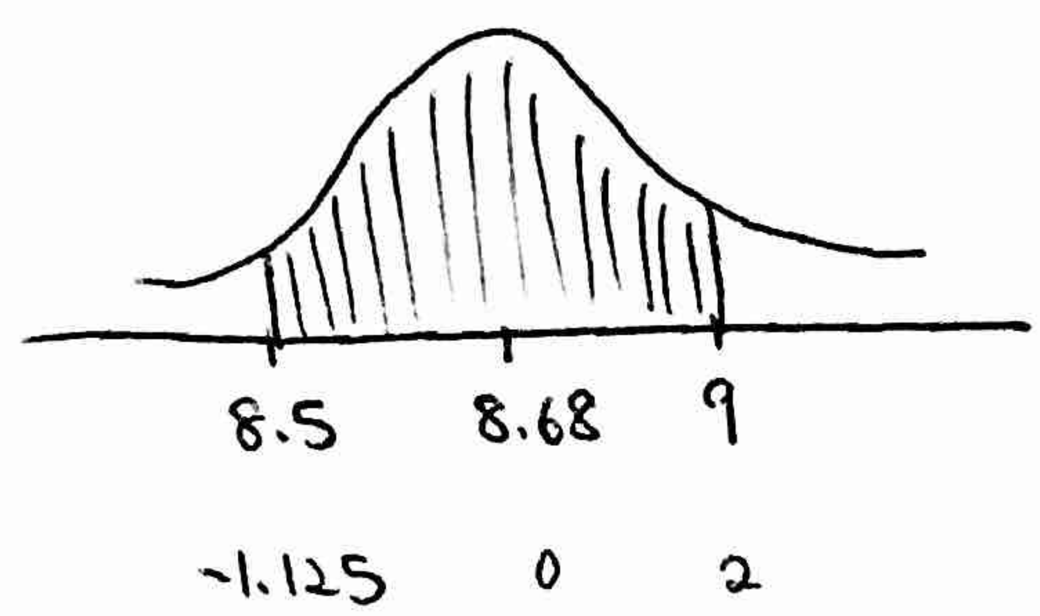
$\Downarrow$

$\boxed{0.0228}$

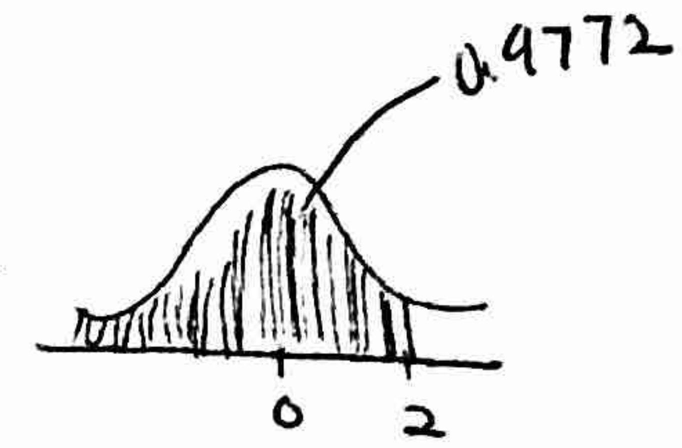
3. Mr. Starnes likes sugar in his hot tea. From experience, he needs between 8.5 and 9 grams of sugar in a cup of tea for the drink to taste right. While making his tea one morning, Mr. Starnes adds four randomly selected packets of sugar. Suppose the amount of sugar in these packets follows a Normal distribution with mean 2.17 grams and standard deviation 0.08 grams. What's the probability that Mr. Starnes's tea tastes right?

$E(S_1+S_2+S_3+S_4) = 4(2.17) = 8.68$ grams

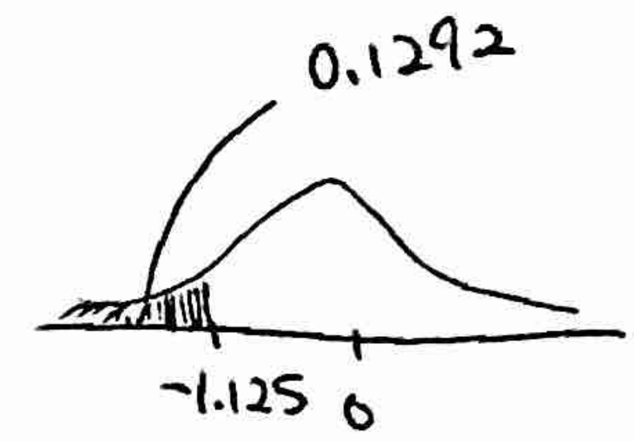
$SD(S_1+S_2+S_3+S_4) = \sqrt{0.08^2(4)} = 0.16$ grams



$z = \dfrac{9-8.68}{0.16} = 2 \Rightarrow$



$z = \dfrac{8.5-8.68}{0.16} = -1.125$



$P(-1.13 < z < 2) = 0.9772 - 0.1292 = 0.848 \approx 0.85$

4. Lamar and Lawrence run a two-person lawn-care service. They have been caring for Mr. Johnson's very large lawn for several years, and they have found that the time it takes Lamar to mow the lawn itself is approximately Normally distributed with a mean of 105 minutes and a standard deviation of 10 minutes. Meanwhile, the time it takes for Lawrence to use the edger and string trimmer to attend to details is also Normally distributed with a mean of 98 minutes and a standard deviation of 15 minutes. They prefer to finish their jobs within 5 minutes of each other. What is the probability that this happens, assuming their finish times are independent?

$D = X - Y$

Lamar $\Rightarrow X$

Lawrence $\Rightarrow Y$

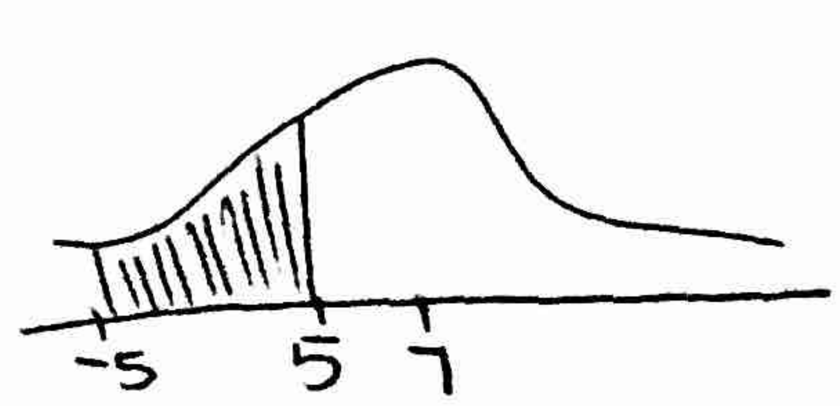
want D to be within 5 mins of each other.

$E(D) = E(X) - E(Y) = 105 - 98 - 7$ mins

$SD(D) = \sqrt{Var(X) + Var(Y)} = \sqrt{10^2+15^2} = 18.03$ mins

$N(7, 18.03)$

$z = \dfrac{-5-7}{18.03} = -0.67 \Rightarrow 0.2514$



$z = \dfrac{5-7}{18.03} = -0.11 \Rightarrow 0.4562$

$P(-0.67 < z < -0.11) \Rightarrow 0.4562 - 0.2514$

$\Rightarrow \boxed{0.2048}$

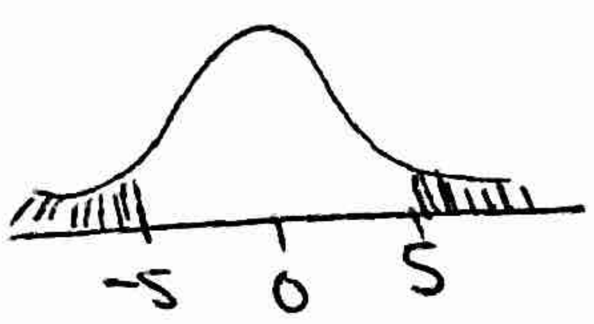
5. **Loser buys the pizza** Leona and Fred are friendly competitors in high school. Both are about to take the ACT college entrance examination. They agree that if one of them scores 5 or more points better than the other, the loser will buy the winner a pizza. Suppose that in fact Fred and Leona have equal ability, so that each score varies Normally with mean 24 and standard deviation 2. (The variation is due to luck in guessing and the accident of the specific questions being familiar to the student.) The two scores are independent. What is the probability that the scores differ by 5 or more points in either direction?

$D = F - L$

$E(D) = E(F) - E(L) = 24 - 24 = 0$

$SD(D) = \sqrt{2^2 + 2^2} = 2.83$

$N(0, 2.83)$



$P(Z \leq -1.77 \text{ or } Z \geq 1.77)$

$= (0.0384) + (0.0384)$

$= \boxed{0.0768}$

$z = \frac{5-0}{2.83} = 1.77 \Rightarrow 0.0384$

$z = \frac{-5-0}{2.83} = -1.77 \Rightarrow 0.0384$

6. The Attila Barbell Company makes bars for weight lifting. The weights of the bars are independent and are normally distributed with a mean of 720 ounces(45 pounds) and a standard deviation of 4 ounces . The bars are shipped 10 in a box to the retailers . The weights of the empty boxes are normally distributed with a mean of 320 ounces and a standard deviation of 8 ounces . The weights of the boxes filled with 10 bars are expected to be normally distributed with a mean of 7,520 ounces. What is the standard deviation of the weights of the boxes filled with 10 bars?

$E(X + Y_1 + Y_2 + Y_3 + \ldots + Y_{10}) = E(X) + E(Y_1) + E(Y_2) + \ldots + E(Y_{10})$

$= 320 + (10)(720) = \boxed{752 \text{ ounces}}$

$SD(T) = \sqrt{8^2 + 4^2(10)} = \boxed{14.97}$

Mixed Practice:

7. Songs on Leila's smartphone have a mean length of 4.3 minutes and a standard deviation of 1.2 minutes. The phone inserts a period of silence between songs that has a mean length of 0.2 minutes and a standard deviation of 0.1 minutes. What are the mean and standard deviation of time it takes for the phone to play a randomly selected two-song sequence?

$E(X_1 + X_2 + Y) = 4.3 + 4.3 + 0.2 = \boxed{8.8 \text{ mins}}$

$SD(X_1 + X_2 + Y) = \sqrt{0.1^2 + 1.2^2 + 1.2^2} = \sqrt{2.89} = \boxed{1.7 \text{ mis}}$

8. **Time and motion** A time-and-motion study measures the time required for an assembly-line worker to perform a repetitive task. The data show that the time required to bring a part from a bin to its position on an automobile chassis varies from car to car according to a Normal distribution with mean 11 seconds and standard deviation 2 seconds. The time required to attach the part to the chassis follows a Normal distribution with mean 20 seconds and standard deviation 4 seconds. The study finds that the times required for the two steps are independent. A part that takes a long time to position, for example, does not take more or less time to attach than other parts.

a) What is the distribution of the time required for the entire operation of positioning and attaching a randomly selected part?

P = positioning time   A = attaching time

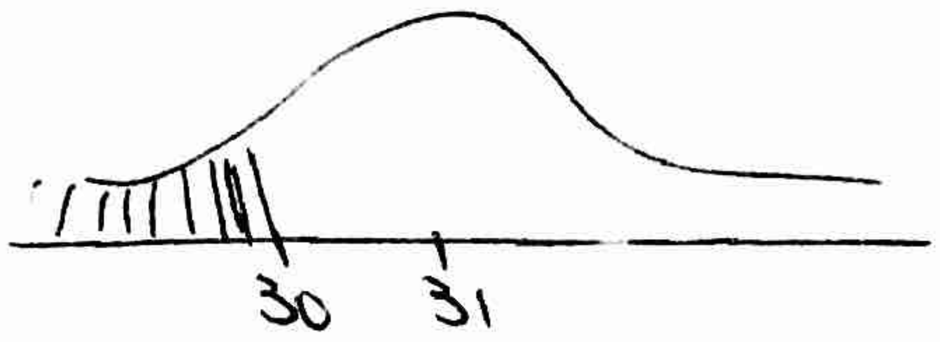
$T = P + A$

$E(T) = 11 + 20 = \boxed{31 \text{ secs}}$

$SD(T) = \sqrt{2^2 + 4^2} = \sqrt{20} = \boxed{4.4721 \text{ secs}}$

b) Management's goal is for the entire process to take less than 30 seconds. Find the probability that this goal will be met for a randomly selected part.

$N(31, 4.47)$



$Z = \frac{30-31}{4.47} = -0.2237$

$\Downarrow$

0.4129

$P(Z < -0.22) = \boxed{0.4129}$

9. The Internal Revenue Service estimates that 8% of all taxpayers filling out long forms make mistakes.

a) An IRS employee starts to randomly select forms—one at a time—to check for mistakes. What is the probability that the first form with mistakes is the 7th one she checks?

$0.92^6 \cdot 0.08 = \boxed{0.485}$

b) The same IRS employee announces at lunch one day that she checked 25 forms this morning and didn't find mistakes on any of them. What is the probability of not getting any mistakes on the first 25 forms?

$0.92^{25} = 0.1244$

10. The number of calories in a 1-ounce serving of a certain breakfast cereal is a random variable with mean 110 and standard deviation 10. The number of calories in a cup of whole milk is a random variable with mean 140 and standard deviation 12. For breakfast, you eat 1 ounce of the cereal with 1/2 cup of whole milk. Let T be the random variable that represents the total number of calories in this breakfast. Find the mean and standard deviation of T.

C = # of calories in cereal (1-ounce)

M = # of calories in a cup of milk

$T = C + 0.5M$

$E(T) = E(C) + 0.5\,E(M) = 110 + 0.5(140) = \boxed{180 \text{ calories}}$

$SD(T) = \sqrt{Var(C + 0.5M)} = \sqrt{Var(C) + 0.5^2\,Var(M)} = \sqrt{10^2 + .5^2(12)^2}$

$= \sqrt{136} = \boxed{11.66 \text{ calories}}$