Key

AIM: How do we use our knowledge of probability to solve various problems? (Day 2) **(4-15E)**

Group Work:

1. A fair coin is tossed four times, and each time the coin lands heads up. If the coin is then tossed 1996 more times, how many heads are most likely to appear in these 1996 additional tosses?

(a) 996
(b) 998
(c) 1000
(d) 1002
(e) 1996

2. A die is loaded so that the number 6 comes up three times as often as any other number. What is the probability of rolling a 1 or a 6?

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

$P(6) = \frac{3}{8}$
$P(1) = \frac{1}{8}$

$\frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

Use the following for questions 3 – 5.
The two-way table below gives information on the performers in the New York Philharmonic Orchestra, categorized by section (type of instrument) and gender.

	Strings	Woodwinds	Brass	Totals
Male	24	8	12	44
Female	37	6	1	44
Totals	61	14	13	88

3. You select one musician from this group at random. What is the probability that this person plays a woodwind?
(a) 0.091 (b) 0.136 (c) 0.159 (d) 0.182 (e) 0.571

$P(w) = \frac{14}{88} \approx 0.159$

4. You select one musician from this group at random. If the person is a male, what is the probability that he plays a woodwind?
(a) 0.091 (b) 0.136 (c) 0.159 (d) 0.182 (e) 0.571

$P(w|m) = \frac{8}{44} = 0.1818$

5. You select one musician from this group at random. Which of the following statement is true about the events "Plays a woodwind" and "Male?"
(a) The events are mutually exclusive and independent.
(b) The events are not mutually exclusive but they are independent.
(c) The events are mutually exclusive, but they are not independent.
(d) The events are not mutually exclusive, nor are they independent.
(e) The events are independent, but we do not have enough information to determine if they are mutually exclusive.

$P(W) \neq P(W|M)$

6. People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population has O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor?

(a) 0
(b) 0.280
(c) 0.526
(d) 0.720
(e) 1

$P(O) = 0.072$

$1 - 0.072 = 0.928$

$1 - P(\text{none type O})$
$= 1 - 0.928^{10} = 0.526$

Use the following for questions ~~8~~ 7 and ~~9~~ 8:

An event A will occur with probability 0.5. An event B will occur with probability 0.6. The probability that both A and B will occur is 0.1.

$P(A) = 0.5$
$P(B) = 0.6$
$P(A \cap B) = 0.1$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.6} = 0.1\bar{7}$

7. The conditional probability of A, given B
(a) is 1/2.
(b) is 3/10.
(c) is 1/5.
(d) is 1/6.
(e) cannot be determined from the information given.

8. We may conclude that

(a) events A and B are independent. x
(b) events A and B are mutually exclusive. x
(c) either A or B always occurs. x
(d) events A and B are complementary. x
(e) none of the above is correct.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\boxed{1} = 0.5 + 0.6 - 0.1$
$P(A \cap B) = P(A) \cdot P(B)$
$0.1 = (0.5)(0.6)$
$0.1 \neq 0.3$

9. If you buy one ticket in the Provincial Lottery, then the probability that you will win a prize is 0.11. Given the nature of lotteries, the probability of winning is independent from month to month. If you buy one ticket each month for five months, what is the probability that you will win at least one prize?

(a) 0.55
(b) 0.50
(c) 0.44
(d) 0.45
(e) 0.56

$1 - P(\text{no price at all})$
$= 1 - (0.89)^5 = 0.44$

10. If P(A) = 0.24 and P(B) = 0.52 and A and B are independent, what is P(A or B)?

(a) 0.1248
(b) 0.28
(c) 0.6352
(d) 0.76
(e) The answer cannot be determined from the information given.

$P(A \cup B) = 0.24 + 0.52 - (0.24)(0.52)$
$= 0.6352$

11. Of people who died in the United States in a recent year, 86% were white, 12% were black, and 2% were Asian. (We will ignore the small number of deaths among other races.) Diabetes caused 2.8% of deaths among whites, 4.4% among blacks, and 3.5% among Asians. The probability that a randomly chosen death was due to diabetes is about

(a) 0.96.
(b) 0.107.
(c) 0.042.
(d) 0.038.
(e) 0.030.

$(0.028)(0.86) + (0.12)(0.044) + (0.02)(0.035)$
$= 0.030$

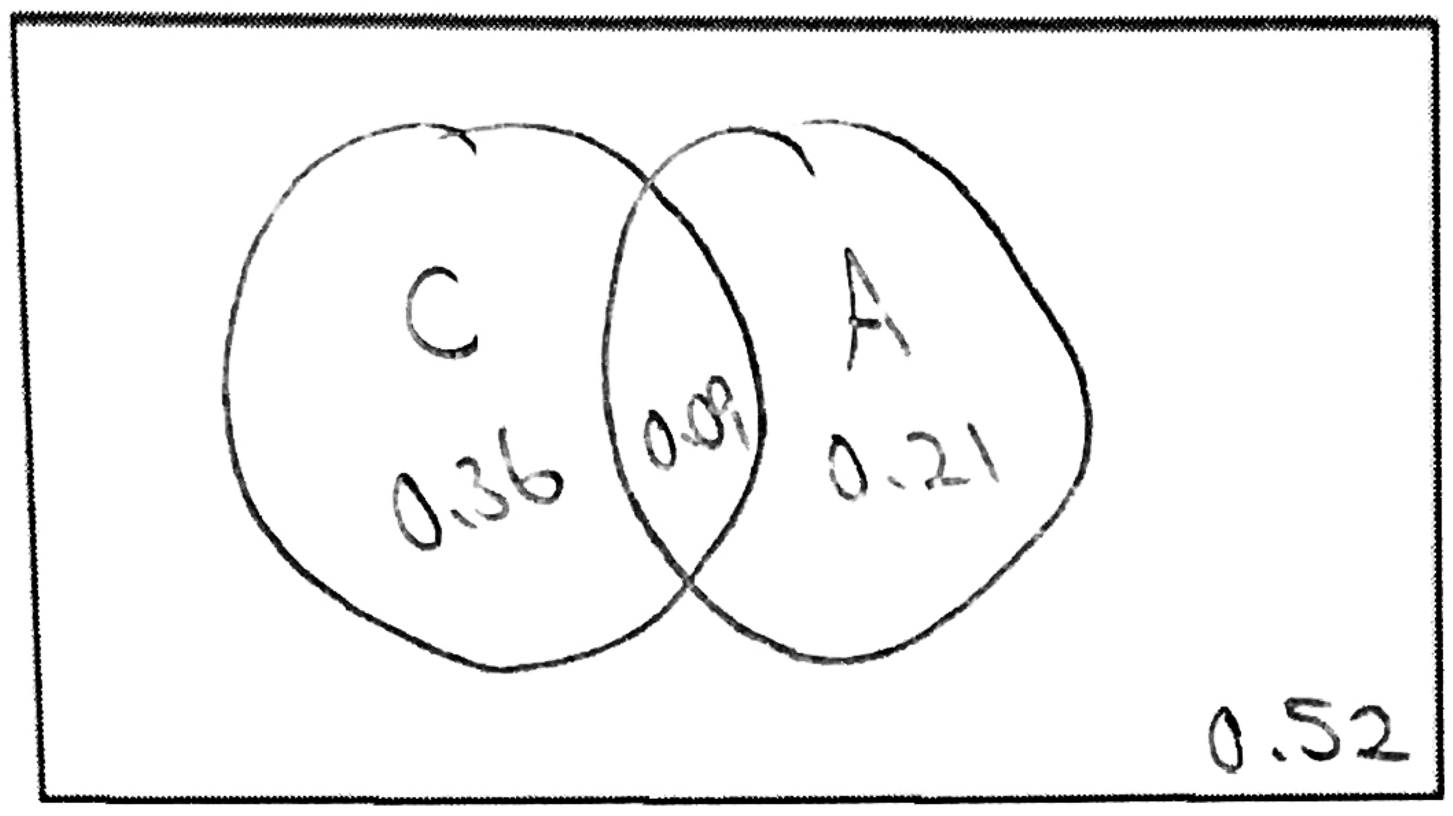
12. In your top dresser drawer are 6 blue socks and 10 grey socks, unpaired and mixed up. One dark morning you pull two socks from the drawer (without replacement, of course!). What is the probability that the two socks match?

(a) 0.075
(b) 0.375
(c) 0.450
(d) 0.500
(e) 0.550

$\frac{6}{16} \cdot \frac{5}{15} + \frac{10}{16} \cdot \frac{9}{15}$
$= 0.50$

13. A grocery store examines its shoppers' product selection and calculates the following: The probability that a randomly-chosen shopper buys apples is 0.21, that the shopper buys potato chips is 0.36, and that the shopper buys both apples and potato chips is 0.09.

(a) Let A = Randomly-chosen shopper buys apples, and C = Randomly-chosen shopper buys potato chips. Sketch a Venn diagram or two-way table that summarizes the probabilities above.



(b) Find each of the following:

i) The probability that a randomly-selected shopper buys apples or potato chips.

$P(A \cup C) = 0.36 + 0.21 - 0.09 = 0.48$

ii) The probability that a randomly-selected shopper buys potato chips or doesn't buy apples.

$P(C \cup A^c) = 0.52 + 0.36$

$= 0.88$

iii) The probability that a randomly-selected shopper doesn't buy apples and doesn't buy potato chips.

$P(A^c \cup C^c) = 0.52$

14. Wile E. Coyote is pursuing the Road Runner across Great Britain toward Scotland. The Road Runner chooses his route randomly, such that there is a probability of 0.8 that he'll take the high road and 0.2 that he'll take the low road. If he takes the high road, the probability that Wile E. catches him is 0.01. If he takes the low road, the probability he gets caught is 0.05. Find the probability that he took the high road, given that he was caught.

$$P(HR \mid \text{Caught}) = \frac{P(HR \cap \text{Caught})}{P(\text{Caught})} = \frac{(0.8)(0.01)}{(0.8)(0.01)+(0.2)(0.05)}$$

$= 0.444$