HW #9

#1

Solution

Part (a):

$P(X>3) = 0.07 + 0.04 + 0.04 + 0.02 = 0.17.$

Part (b):

Y = number of households in violation.

Y has a binomial distribution with $n = 10$ and $p = 0.17$.

$$P(Y=2) = \binom{10}{2}(0.17)^2(0.83)^8 = 0.2929.$$

Part (c):

The distribution of $\bar{X}$ will:

1. be approximately normal;
2. have mean $\mu_{\bar{X}} = \mu = 1.65$;
3. have standard deviation $\sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.851}{\sqrt{150}} = 0.1511.$

#2

Solution

Part (a):

X is normally distributed with $\mu = 170$ and $\sigma = 20$, and Y is normally distributed with $\mu = 200$ and $\sigma = 10$.
The distribution of $Y - X$ has mean and standard deviation:

$$\mu_{Y-X} = \mu_Y - \mu_X = 200 - 170 = 30$$
$$\sigma_{Y-X} = \sqrt{\sigma_Y^2 + \sigma_X^2} = \sqrt{10^2 + 20^2} = \sqrt{500} = 22.36$$

The distribution of $Y - X$ is normal with mean 30 and standard deviation 22.36 (or, variance 500).

Part (b):

The train from Bullsnake will have to wait when $Y - X$ is positive:

$$P(Y - X > 0) = P\left(z > \frac{0-30}{22.36}\right) = P(z > -1.34) = 0.9099$$

(Calculator: 0.9082408019 or 0.9078172963, if z is not rounded.)
The proportion of days that the train will have to wait is about 0.91.

Part (c):

Let D denote the delay that will be needed for the train leaving Bullsnake. With the additional constant delay,

$$X + D \text{ is normally distributed with } \mu_{X+D} = 170 + D \text{ and } \sigma_{X+D} = \sigma_X = 20$$
$$Y \text{ is normally distributed with } \mu_Y = 200 \text{ and } \sigma_Y = 10$$

Thus, the difference $Y - (X + D)$ is normally distributed with

$$\mu_{Y-(X+D)} = \mu_Y - \mu_{(X+D)} = 200 - (D + 170) = 30 - D$$
$$\sigma_{Y-(X+D)} = \sigma_{Y-X} = 22.36$$

The combined delay and travel time ($X + D$) for the Bullsnake train must be *less than* the travel time for the Diamondback train (Y) with probability 0.01. That is, $P(Y - (X + D) > 0) = 0.01$, so we need

$$\frac{0 - \mu_{Y-(X+D)}}{\sigma_{Y-(X+D)}} = \frac{0 - (30 - D)}{22.36} = 2.33$$

Solving for D, the train from Bullsnake should be delayed by 82.099 minutes.