Name/Date: Key **AP Statistics Unit 4** **HW #4.17D**

1. **10% condition** To use a binomial distribution to approximate the count of successes in an SRS, why do we require that the sample size *n* be no more than 10% of the population size N?

If the sample is a small fraction of the population (less than 10%), the make up of the population doesn't change enough to make the lack of independent trials an issue.

2. ***On the Web** What kinds of Web sites do males aged 18 to 34 visit most often? Half of male Internet users in this age group visit an auction site such as eBay at least once a month. A study of Internet use interviews a random sample of 500 men aged 18 to 34. Let X = the number in the sample who visit an auction site at least once a month.

a) Show that X is approximately a binomial random variable.

B? Success - visit an auction site an least once a month; failure - don't visit an auction site at least once a month

I? we are sampling without replacement but the sample size is far less than 10% of all males aged 18-34 (n=500).

N? n = 500

S? p = 1/2

b) Use a Binomial distribution to find the probability that at least 235 of the men in the sample visit an online auction site at least once a month.

10% is satisfied - ok to use binomial distribution

P (X greater than and equal to 235) = 1 - P(X less than and equal to 234)

= 1 - binomcdf (500, 1/2, 234) = 0.9172

c) Check the conditions for using a Normal approximation in this setting.

Large Counts Condition

$np = (500)(1/2) = 250$

& $n(1-p) = (500)(\frac{1}{2}) = 250$

are both at least 10

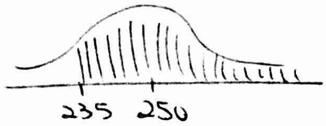
Conditions are satisfied. we can use the Normal approximation.

d) Use a Normal distribution to estimate the probability that at least 235 of the men in the sample visit an online auction site at least once a month.

$\mu_x = np = (500)(1/2) = 250$ $\qquad P(X \geq 235)$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{500(1/2)(1/2)} = 11.18$

$\sim N(250, 11.18)$



$Z = \frac{235-250}{11.18} = -1.34$

$\Rightarrow \boxed{0.9099}$

3. ***Checking for survey errors** One way of checking the effect of undercoverage, nonresponse, and other sources of error in a sample survey is to compare the sample with known facts about the population. About 12% of American adults identify themselves as black. Suppose we take an SRS of 1500 American adults and let X be the number of blacks in the sample.

a) Show that X is approximately a binomial random variable.

B? Success - person identifies themselves as black; failure - person does not

I? we are sampling w/o replacement. But the sample size 1500 is far less than 10% of all American adults

N? $n = 1500$

S? $p = 0.12$

b) Check the conditions for using a Normal approximation in this setting.

$np = 1500(0.12) = 180$

$n(1-p) = 1500(0.88) = 1320$

Both are at least 10.

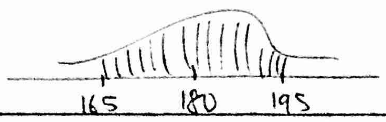
Conditions met → Safe to use Normal approximation.

c) Use a Normal distribution to estimate the probability that the sample will contain between 165 and 195 blacks.

$\mu_x = np\ (1500)(0.12) = 180$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{1500(0.12)(0.88)} = 12.5857$

$\sim N(180, 12.5857)$



$P(165 \leq X \leq 195) = P(-1.19 \leq Z \leq 1.19)$

$= P(Z \leq 1.19) - P(Z \leq -1.19)$

$= 0.8830 - 0.1170 = \boxed{0.7660}$

normal cdf (lower: 165, upper: 195, μ: 180, σ: 12.5857) $= \boxed{0.7667}$

Mixed Review

4. A fast-food restaurant runs a promotion in which certain food items come with game pieces. According to the restaurant, 1 in 4 game pieces is a winner.

a) If Jeff gets 4 game pieces, what is the probability that he wins exactly 1 prize? Binomial

$P(X=1)$

$\binom{4}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^3 = 0.4219$

binompdf (4, 1/4, 1)

b) If Jeff keeps playing until he wins a prize, what is the probability that he has to play the game exactly 5 times? Geometric

$P(X=5) = (0.75)^4(0.25)$

$= 0.0791$

Geompdf (0.25, 5)

5. *In which of the following situations would it be appropriate to use a Normal distribution to approximate probabilities for a binomial distribution with the given values of n and p?

(a) $n = 10, p = 0.5$
(b) $n = 40, p = 0.88$
(c) $n = 100, p = 0.2$
(d) $n = 100, p = 0.99$
(e) $n = 1000, p = 0.003$

$\rightarrow np \geq 10$
$\rightarrow n(1-p) \geq 10$

6. According to Mars, Incorporated, 20% of its plain M&M'S candies are orange. Assume that the company's claim is true. Suppose that you reach into a large bag of plain M&M'S (without looking) and pull out 8 candies. Let X = the number of orange candies you get. → 1000 M&Ms

a) Explain why it is reasonable to use the binomial distribution for probability calculations involving X.

B? success – orange ; failure – not orange

I? not quite – check 10% condition n=8 less than 10% of the large bag.

N? n=8

S? p=0.2

b) Find and interpret the expected value of X.

$\mu_X = 8(0.2) = 1.6$

If were to select many samples of size 8, we ~~should~~ would expect to get about 1.6 orange M&M's on average.

c) Find and interpret the standard deviation of X.

$\sigma_X = \sqrt{8(0.2)(0.8)} = 1.13$

If we were to select many samples of size 8, the # of orange M&Ms would typically vary by about 1.13 from the mean.

d) Would you be surprised if none of the candies were orange? Compute an appropriate probability to support your answer.

$P(X=0) = \binom{8}{0}(0.2)^0(0.8)^8 = 0.1678$

probability is not that small.

It would not be surprising.