Name/Date: Key AP Statistics Unit 4 HW #4.16B

1. **Random variables.** Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of:

	Mean	SD
X	80	12
Y	12	3

a) X – 20

$E(X-20) = E(X) - 20 = 80 - 20 = \boxed{60}$

$SD(X-20) = SD(X) = \boxed{12}$

b) 0.5Y

$E(0.5Y) = 0.5\,E(Y) = 0.5(12) = \boxed{6}$

$SD(0.5Y) = \sqrt{0.5^2\,SD(Y)} = \sqrt{0.25(9)} = \boxed{1.5}$

b) X + Y

$E(X+Y) = E(X) + E(Y) = 80 + 12 = \boxed{92}$

$SD(X+Y) = \sqrt{Var(X+Y)} = \sqrt{Var X + Var Y}$

$= \sqrt{12^2 + 3^2} = \boxed{12.37}$

c) X – Y

$E(X-Y) = E(X) - E(Y) = 80 - 12 = \boxed{68}$

$SD(X-Y) = 12.37$

d) $Y_1 + Y_2$

$E(Y_1 + Y_2) = E(Y_1) + E(Y_2) = 12 + 12 = 24$

$SD(Y_1 + Y_2) = \sqrt{Var(Y_1) + Var(Y_2)} = \sqrt{3^2 + 3^2} = \sqrt{18} = \boxed{4.24}$

2. The American Veterinary Association claims that the annual cost of medical care for dogs averages \$100 with a standard deviation of \$30, and for cats averages \$120 with a standard deviation of \$35.

D = cost for dogs C = cost for cats

a) Find the expected value for the annual cost of medical care for a person who has one dog and one cat.

$E(D+C) = E(D) + E(C) = 100 + 120 = \boxed{\$220}$

b) Find the standard deviation for the annual cost of medical care for a person who has one dog and one cat.

$SD(D+C) = \sqrt{Var D + Var C}$

$= \sqrt{30^2 + 35^2} = \sqrt{2125} = \boxed{\$46.10}$

c) Suppose that a couple owns *four* dogs.

i) Find the expected value for the annual cost of medical care for the couple's dogs.

$E(D_1 + D_2 + D_3 + D_4) = 100 + 100 + 100 + 100$

$= \boxed{\$400}$

ii) Find the standard deviation for the annual cost of medical care for the couple's dogs.

$SD(D_1 + D_2 + D_3 + D_4)$

$= \sqrt{Var(D_1) + Var(D_2) + Var(D_3) + Var(D_4)}$

$= \sqrt{30^2 + 30^2 + 30^2 + 30^2} = \sqrt{3600}$

$= \boxed{\$60}$

3. **Eggs.** A grocery supplier believes that in a dozen eggs, the mean number of broken ones is 0.6 with a standard deviation of 0.5 eggs. You buy 3 dozen eggs without checking them.

X = broken eggs in dozen egg

a) How many broken eggs do you expect to get?

$E(X_1+X_2+X_3) = E(X_1)+E(X_2)+E(X_3)$

$= 0.6+0.6+0.6 = 1.8$ eggs

b) What's the standard deviation?

$SD(X_1+X_2+X_3)$

$= \sqrt{0.5^2+0.5^2+0.5^2} \approx 0.87$ eggs

c) What assumptions did you have to make about the eggs in order to answer this question?

The cartons of eggs must be independent of each other.

4. **Donations.** Organizers of a televised fundraiser know from past experience that most people donate small amounts (\$10–\$25), some donate larger amounts (\$50–\$100), and a few people make very generous donations of \$250, \$500, or more. Historically, pledges average about \$32 with a standard deviation of \$54. If 120 people call in pledges, what are the mean and standard deviation of the total amount raised?

$E(X_1+X_2+\dots X_{120}) = E(X_1)+E(X_2)+\dots+E(X_{120})$

$= 32+32+\dots+32 = (32)(120) = \$3840$

$SD(X_1+X_2+\dots+X_{120}) = \sqrt{54^2+54^2\dots+54^2}$

$= \sqrt{(54^2)(120)} \approx \$591.54$

5. The manager of a children's puppet theatre has determined that the number of adult tickets he sells for a Saturday afternoon show is a random variable with a mean of 28.3 tickets and a standard deviation of 5.3 tickets. The mean number of children's tickets he sells is 42.5, with a standard deviation of 8.1.

X = # of tickets (adult)

Y = # of tickets (children)

a) The adult tickets sell for \$10. Let A = the money he collects from adult tickets on a random Saturday. What are the mean and standard deviation of A?

$A = 10X$

$E(A) = E(10X) = 10\,E(X) = 10(28.3)$

$= \$283$

$SD(10X) = \sqrt{10^2 \operatorname{var} X} = \sqrt{100(5.3)^2}$

$= \$53$

b) The children's tickets sell for \$6. Let T = the money he collects from all ticket sales (adults and children) on a random Saturday. Assume (unrealistically, perhaps) that the number of tickets sold to adults is independent of the number sold to children. What are the mean and standard deviation of T?

$T = 10X + 6Y$

$E(T) = E(10X+6Y) = 10E(X)+6E(Y)$

$= 10(28.3)+6(42.5)$

$= \$538$

$SD(10X+6Y) = \sqrt{10^2 \operatorname{var} X + 6^2 \operatorname{var} Y}$

$= \sqrt{100(5.3)^2+36(8.1)^2}$

$\approx \$71.91$