HW 5.19A answers

1.

. Cloning 2007.

a) $ME = z^* \times SE(\hat{p}) = z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.960 \times \sqrt{\frac{(0.11)(0.89)}{1003}} \approx 1.9\%$

b)

A 90% confidence interval results in a smaller margin of error. If confidence is decreased, a smaller interval is constructed.

c)

$$ME = z^* \times SE(\hat{p}) = z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \times \sqrt{\frac{(0.11)(0.89)}{1003}} \approx 1.6\%$$

2.

. Take the offer.

a) Randomization condition: Offers were sent to a random sample of 50,000 cardholders.
10% condition: We assume that there are more than 500,000 cardholders.
Success/Failure condition: $n\hat{p} = 1{,}184$ and $n\hat{q} = 48{,}816$ are both much greater than 10, so the sample is large enough.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{1{,}184}{50{,}000}\right) \pm 1.960\sqrt{\frac{\left(\frac{1{,}184}{50{,}000}\right)\left(\frac{48{,}816}{50{,}000}\right)}{50{,}000}} = (0.0223, 0.0250)$$

We are 95% confident that the between 2.23% and 2.5% of all cardholders would register for double miles.

3.

. Teenage drivers.

a) Independence assumption: There is no reason to believe that accidents selected at random would be related to one another.
Randomization condition: The insurance company randomly selected 582 accidents.
10% condition: 582 accidents represent less than 10% of all accidents.
Success/Failure condition: $n\hat{p} = 91$ and $n\hat{q} = 491$ are both greater than 10, so the sample is large enough.

Since the conditions are met, we can use a one-proportion *z*-interval to estimate the percentage of accidents involving teenagers.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{91}{582}\right) \pm 1.960\sqrt{\frac{\left(\frac{91}{582}\right)\left(\frac{491}{582}\right)}{582}} = (12.7\%, 18.6\%)$$

b) We are 95% confident that between 12.7% and 18.6% of all accidents involve teenagers.

c) About 95% of random samples of size 582 will produce intervals that contain the true proportion of accidents involving teenagers.

4.

Junk mail.

a) **Independence assumption:** There is no reason to believe that one randomly selected person's response will affect another's.
Randomization condition: The company randomly selected 1000 recipients.
10% condition: 1000 recipients is less than 10% of the population of 200,000 people.
Success/Failure condition: $n\hat{p} = 123$ and $n\hat{q} = 877$ are both greater than 10, so the sample is large enough.

Since the conditions are met, we can use a one-proportion *z*-interval to estimate the percentage of people who will respond to the new flyer.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{123}{1000}\right) \pm 1.645 \sqrt{\frac{\left(\frac{123}{1000}\right)\left(\frac{877}{1000}\right)}{1000}} = (10.6\%, 14.0\%)$$

b) We are 90% confident that between 10.6% and 14.0% of people will respond to the new flyer.

c) About 90% of random samples of size 1000 will produce intervals that contain the true proportion of people who will respond to the new flyer.