Name: Key Unit 4 - Review Packet (Ch. 14 – 17)
(WILL BE COUNTED AS 3 HW ASSIGNMENT GRADE!!!)

Section I: Multiple Choice Select the best answer for each question.

Questions 1 to 3 refer to the following setting. A psychologist studied the number of puzzles that subjects were able to solve in a five-minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a randomly chosen subject. The psychologist found that X had the following probability distribution:

Value:	1	2	3	4
Probability:	0.2	0.4	0.3	0.1

1. What is the probability that a randomly chosen subject completes more than the expected number of puzzles in the five-minute period while listening to soothing music?

Find $E(X) = (1)(0.2) + (2)(0.4) + 3(0.3) + 4(0.1)$
$= 2.3$

(a) 0.1
(b) 0.4
(c) 0.8
(d) 1
(e) Cannot be determined

$P(X > 2.3) = P(X=3) + P(X=4)$
$= 0.3 + 0.1$
$= \boxed{0.4}$

2. The standard deviation of X is 0.9. Which of the following is the best interpretation of this value?

(a) About 90% of subjects solved 3 or fewer puzzles.
(b) About 68% of subjects solved between 0.9 puzzles less and 0.9 puzzles more than the mean.
(c) The typical subject solved an average of 0.9 puzzles.
(d) The number of puzzles solved by subjects typically differed from the mean by about 0.9 puzzles.
(e) The number of puzzles solved by subjects typically differed from one another by about 0.9 puzzles.

3. Let D be the difference in the number of puzzles solved by two randomly selected subjects in a five-minute period. What is the standard deviation of D?

$SD(X_1 - X_2) = \sqrt{Var(X_1 - X_2)}$
$= \sqrt{Var(X_1) + Var(X_2)}$
$= \sqrt{0.9^2 + 0.9^2} = 1.27$

(a) 0
(b) 0.81
(c) 0.9
(d) 1.27
(e) 1.8

4. Suppose a student is randomly selected from your school. Which of the following pairs of random variables are most likely independent? one doesn't affect the other.

(a) X = student's height; Y = student's weight
(b) X = student's IQ; Y = student's GPA
(c) X = student's PSAT Math score; Y = student's PSAT Verbal score
(d) X = average amount of homework the student does per night; Y = student's GPA
(e) X = average amount of homework the student does per night; Y = student's height

5. A certain vending machine offers 20-ounce bottles of soda for \$1.50. The number of bottles X bought from the machine on any day is a random variable with mean 50 and standard deviation 15. Let the random variable Y equal the total revenue from this machine on a given day. Assume that the machine works properly and that no sodas are stolen from the machine. What are the mean and standard deviation of Y?

(a) $\mu_Y = \$1.50$, $\sigma_Y = \$22.50$
(b) $\mu_Y = \$1.50$, $\sigma_Y = \$33.75$
(c) $\mu_Y = \$75$, $\sigma_Y = \$18.37$
(d) $\mu_Y = \$75$, $\sigma_Y = \$22.50$
(e) $\mu_Y = \$75$, $\sigma_Y = \$33.75$

$Y = 1.5X$

$E(Y) = 1.5E(X) = 1.5(50) = \boxed{75}$

$\sigma_Y = SD(Y) = \sqrt{Var(1.5X)}$

$= \sqrt{1.5^2 Var(X)} = \sqrt{1.5^2 (15)^2} = \boxed{22.5}$

6. The weight of tomatoes chosen at random from a bin at the farmer's market follows a Normal distribution with mean $\mu = 10$ ounces and standard deviation $\sigma = 1$ ounce. Suppose we pick four tomatoes at random from the bin and find their total weight T. The random variable T is

~~(a)~~ Normal, with mean 10 ounces and standard deviation 1 ounce.
(b) Normal, with mean 40 ounces and standard deviation 2 ounces.
~~(c)~~ Normal, with mean 40 ounces and standard deviation 4 ounces.
~~(d)~~ binomial, with mean 40 ounces and standard deviation 2 ounces.
~~(e)~~ binomial, with mean 40 ounces and standard deviation 4 ounces.

$T = X_1 + X_2 + X_3 + X_4$

$E(T) = E(X) \cdot 4 = \boxed{40}$

$SD(T) = \sqrt{Var(T)} = \sqrt{Var(X) + Var(X) + Var(X) + Var(X)}$

$= \sqrt{(1)^2 + 1^2 + 1^2 + 1^2}$

$= \sqrt{4} = \boxed{2}$

7. Which of the following random variables is geometric?

• independent
• until get a success

(a) The number of times I have to roll a die to get two 6s. ✗
(b) The number of cards I deal from a well-shuffled deck of 52 cards until I get a heart. ✗ not independent
(c) The number of digits I read in a randomly selected row of the random digits table until I find a 7. ✓
(d) The number of 7s in a row of 40 random digits. ✗
(e) The number of 6s I get if I roll a die 10 times. ✗

8. Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?

Binomial w/ parameters $n = 17$; $p = 0.4$

~~(a)~~ 0.011
(b) 0.035
~~(c)~~ 0.092
(d) 0.965
(e) 0.989

$P(X > 10) = 1 - P(X \le 10)$

$= 1 - \text{binomcdf}(17, 0.4, 10)$

$= \boxed{0.0348}$

9. A test for extrasensory perception (ESP) involves asking a person to tell which of 5 shapes—a circle, star, triangle, diamond, or heart—appears on a hidden computer screen. On each trial, the computer is equally likely to select any of the 5 shapes. Suppose researchers are testing a person who does not have ESP and so is just guessing on each trial. What is the probability that the person guesses the first 4 shapes incorrectly but gets the fifth correct?

$\left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)$

(a) 1/5

(b) $\left(\frac{4}{5}\right)^4$

(c) $\left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)$

(d) $\binom{5}{1} \cdot \left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)$

(e) 4/5

10. BatCo, a company that sells batteries, claims that 99.5% of their batteries are in working order. How many would you expect to buy, on average, to find one that does not work?

— Geometric

(a) 5
(b) 100
(c) 200
(d) 995
(e) 2000

p(success) ⇒ 0.5% not working

$E(x) = \frac{1}{p} = \frac{1}{0.005} = 200$

Section II: Free Response Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

11. Let Y denote the number of broken eggs in a randomly selected carton of one dozen "store brand" eggs at a local supermarket. Suppose that the probability distribution of Y is as follows.

Value y_i:	0	1	2	3	4
Probability p_i:	0.78	0.11	0.07	0.03	0.01

a) What is the probability that at least 10 eggs in a randomly selected carton are unbroken?

means 10, 11, 12 unbroken

⇒ 2, 1, 0 broken

$P(X=2) + P(X=1) + P(X=0) = 0.78 + 0.11 + 0.07 = \boxed{0.96}$

b) Calculate and interpret μ_Y,

$\mu_Y = (0)(0.78) + (1)(0.11) + 2(0.07) + 3(0.03) + 4(0.01)$

$= 0.38$

If we were to randomly select many cartons of eggs, we would expect to find about 0.38 broken eggs.

c) Calculate and interpret σ_Y. Show your work.

$\sigma_Y = SD(Y) = \sqrt{(0-0.38)^2(0.78) + (1-0.38)^2(0.11) + (2-0.38)^2(0.07) + \dots + (4-0.38)^2(0.01)}$

$= \sqrt{0.6756} = 0.8219$

The number of broken eggs would typically vary from the mean by about 0.8219.

d) A quality control inspector at the store keeps looking at randomly selected cartons of eggs until he finds one with at least 2 broken eggs. Find the probability that this happens in one of the first three cartons he inspects.

→ Geometric

P(success)

= P(at least 2 broken eggs)

$= 0.07 + 0.03 + 0.01 = 0.11$ ↙ P(success)

$P(X \le 3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.11 + (0.89)(0.11) + (0.89)^2(0.11) = \boxed{0.2950}$

OR

Geomecdf (P: 0.11, x-value: 3)

$= \boxed{0.2950}$

12. Ed and Adelaide attend the same high school, but are in different math classes. The time E that it takes Ed to do his math homework follows a Normal distribution with mean 25 minutes and standard deviation 5 minutes. Adelaide's math homework time A follows a Normal distribution with mean 50 minutes and standard deviation 10 minutes. Assume that E and A are independent random variables.

a) Randomly select one math assignment of Ed's and one math assignment of Adelaide's. Let the random variable D be the difference in the amount of time each student spent on their assignments: D = A – E. Find the mean and the standard deviation of D. Show your work.

$$E(D) = E(A-E) = E(A) - E(E) = 50-25 = \boxed{25 \text{ mins}}$$

$$SD(D) = \sqrt{Var(A-E)} = \sqrt{Var(A)+Var(E)}$$

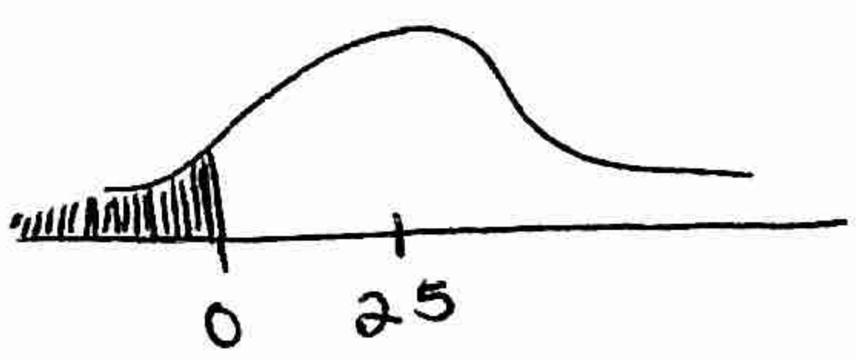
$$= \sqrt{5^2+10^2} = \boxed{11.18 \text{ mins}}$$

b) Find the probability that Ed spent longer on his assignment than Adelaide did on hers. Show your work.

D ⊖ $\quad D<0$

we want $P(A<E) = P(D<0)$

$\sim N(25, 11.18)$



$$Z = \frac{0-25}{11.18} = -2.24$$

$$P(Z<-2.24) = \boxed{0.0125}$$

There is a 0.0127 probability that ED spent longer on his assignment than Adelaide did on hers.

13. According to the Census Bureau, 13% of American adults (aged 18 and over) are Hispanic. An opinion poll plans to contact an SRS of 1200 adults.

Binomial w/ $n = 1200$; $p = 0.13$ $\quad X$ = # of Hispanics in the sample.

a) What is the mean number of Hispanics in such samples? What is the standard deviation?

$$\mu_x = np = (1200)(0.13) = \boxed{156}$$

$$\sigma_x = \sqrt{np(1-p)} = \sqrt{2000(0.13)(0.87)} = \boxed{11.6499}$$

b) Should we be suspicious if the sample selected for the opinion poll contains 15% Hispanic people? Compute an appropriate probability to support your answer.

15% ⇒ 1200(0.15) = 180 Hispanics

We want to find $P(X \geq 180) = 1 - \text{binomcdf}(1200, 0.13, 179)$

$= 1 - 0.9765$

$= \boxed{0.0235}$

OR we can use Binomial approximation.

14. Luxury cars According to *infoplease*, 18.8% of the luxury cars manufactured in 2003 were silver. A large car dealership typically sells 50 luxury cars a month.

a) Explain why you think that the luxury car sales can be considered Bernoulli trials.

Note: Bernoulli trials, if . . .
1. There are two possible outcomes.
2. The probability of success is constant.
3. The trials are independent.
Geometric and Binomial model are Bernoulli trials.

① success - silver ; failure - other than silver

② Trials are not independent, but 10% condition is satisfied. (50 cars are less than 10% of all luxuary cars sold in the Country)

③ p(success) is constant.

b) What is the probability that the fifth luxury car sold is the first silver one?

Geometric $p = 0.188$; $q = 0.812$

$P(X=5) = (0.812)^4 (0.188) = \boxed{0.817}$

c) Let X represent the number of silver luxury cars sold in a typical month. What is the probability model for X? Specify the model (name and parameters), and tell the mean and standard deviation.

Binomial w/ parameters $n = 50$, $p = 0.188$

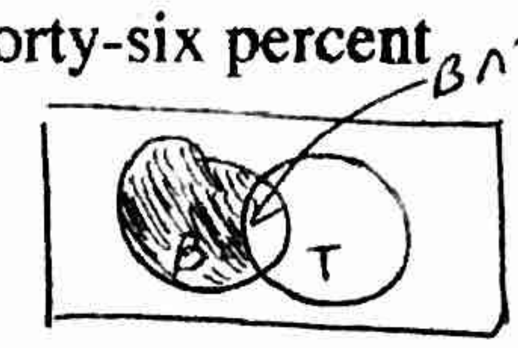
$\mu_x = np = (50)(0.188) = \boxed{9.4}$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{50(0.188)(0.812)} = \sqrt{7.6328}$

$= \boxed{2.76}$

Use the following information for questions 15-16:
In an AP Stats class, 57% of students eat breakfast in the morning and 80% of students floss their teeth. Forty-six percent of students eat breakfast and also floss their teeth.

$P(B \cap T) = 0.46 \quad P(B) = 0.57 \quad P(T) = 0.8$



15. What is the probability that a student from this class eats breakfast but does not floss their teeth?
A) 9% B) 11% C) 34% D) 57% E) 91%

$P(B) - P(B \cap T) = 0.57 - 0.46 = 0.11$

16. What is the probability that a student from this class eats breakfast or flosses their teeth?
A) 9% B) 11% C) 34% D) 57% E) 91%

$P(B \cup T) = P(B) + P(T) - P(B \cap T)$
$= 0.57 + 0.8 - 0.46 = 0.91$

17. Five juniors and four seniors have applied for two open student council positions. School administrators have decided to pick the two new members randomly. What is the probability they are both juniors or both seniors?
A) 0.395 B) 0.444 C) 0.506 D) 0.569 E) 0.722

$\left(\frac{5}{9}\right)\left(\frac{4}{8}\right) + \left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = 0.4\overline{4}$

18. A fair coin has come up "heads" 10 times in a row. The probability that the coin will come up heads on the next flip is
A) less than 50%, since "tails" is due to come up.
B) 50%.
C) greater than 50%, since it appears that we are in a streak of "heads."
D) It cannot be determined.

19. According to the National Telecommunication and Information Administration, 56.5% of U.S. households owned a computer in 2001. What is the probability that of three randomly selected U.S. households at least one owned a computer in 2001?
A) 18.0% B) 43.5% C) 56.5% D) 82.0% E) 91.8%

$1 - P(\text{none at all}) = 1 - (0.435)^3$
$= 0.917687$

20. According to the National Telecommunication and Information Administration, 50.5% of U.S. households had Internet access in 2001. What is the probability that four randomly selected U.S. households all had Internet access in 2001?
A) 6.5% B) 12.6% C) 49.5% D) 50.5% E) 93.5%

$(0.505)^4 = 0.065$

21. Some marathons allow two runners to "split" the marathon by each running a half marathon. Alice and Sharon plan to split a marathon. Alice's half-marathon times average 92 minutes with a standard deviation of 4 minutes, and Sharon's half-marathon times average 96 minutes with a standard deviation of 2 minutes. Assume that the women's half-marathon times are independent. The expected time for Alice and Sharon to complete a full marathon is 92 + 96 = 188 minutes. What is the standard deviation of their total time?
A) 2 minutes
B) 4.5 minutes
C) 6 minutes
D) 20 minutes
E) It cannot be determined

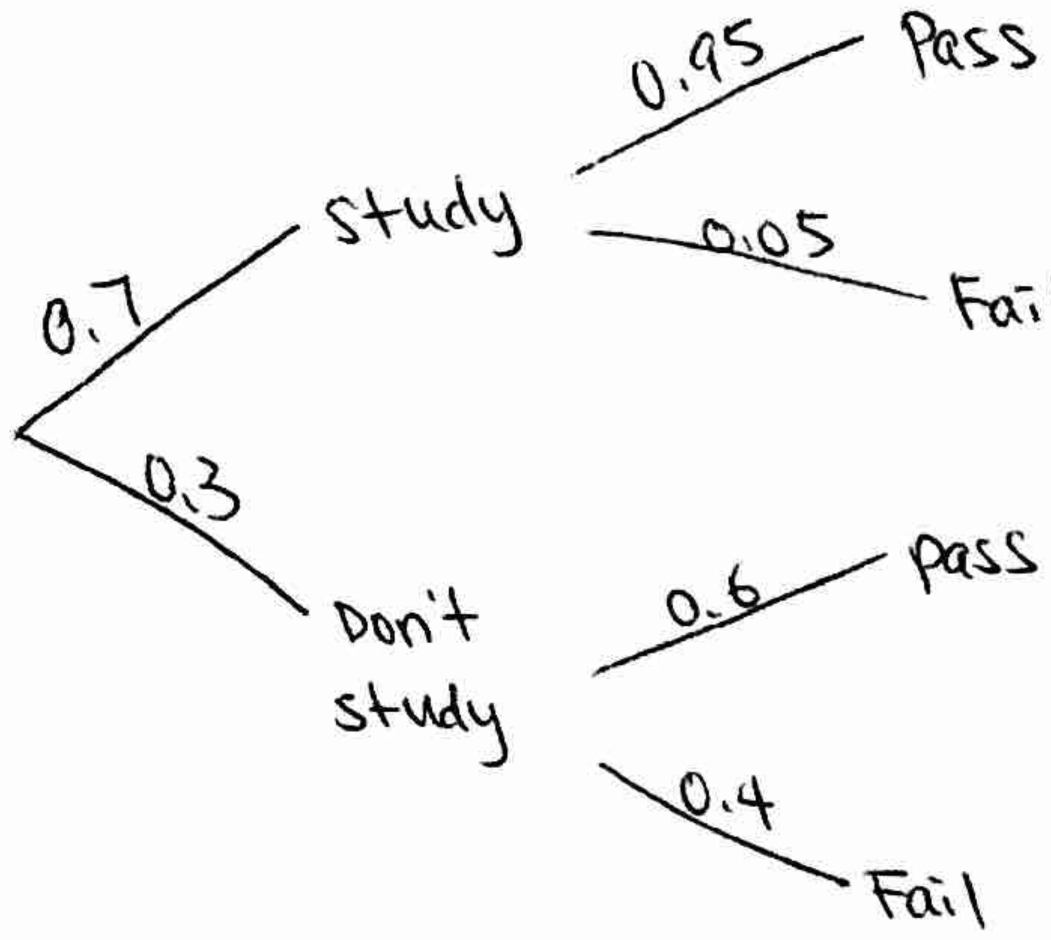
$A \Rightarrow \mu_A = 92;\ \sigma_A = 4$

$S \Rightarrow \mu_S = 96;\ \sigma_S = 2$

$T = A + S$

$SD(T) = \sqrt{Var(A+S)}$
$= \sqrt{Var(A) + Var(S)} = \sqrt{4^2 + 2^2}$
$= 4.47$

22. **Passing the test** Assume that 70% of teenagers who go to take the written drivers license test have studied for the test. Of those who study for the test, 95% pass; of those who do not study for the test, 60% pass. What is the probability that a teenager who passes the written drivers license test did not study for the test?



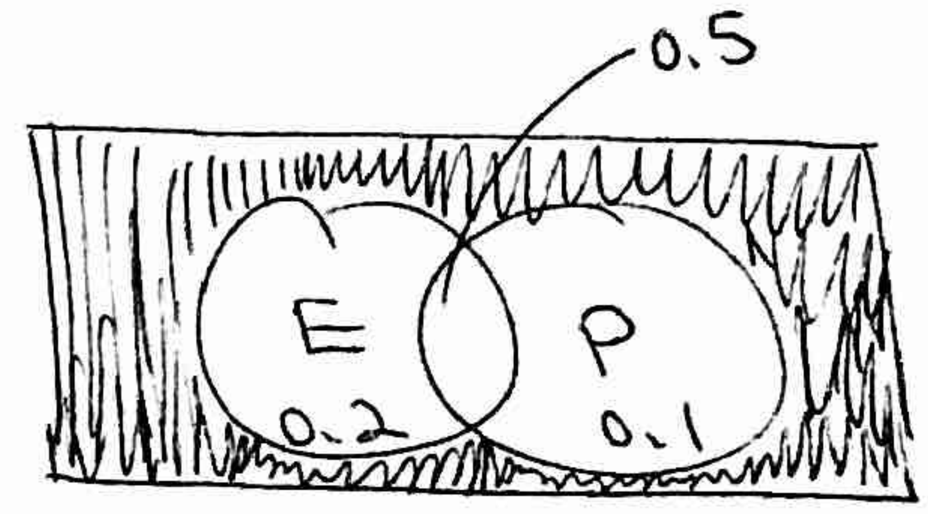
$$P(\text{NOT study} \mid \text{pass}) = \frac{P(\text{didn't study} \cap \text{pass})}{P(\text{pass})}$$

$$= \frac{(0.3)(0.6)}{(0.7)(0.95)+(0.3)(0.6)}$$

$$= \frac{0.18}{0.845} = \boxed{0.2130}$$

23. **Grades** You believe that there is a 20% chance that you will earn an A in your English class, a 10% chance that you will earn an A in your Physics class, and a 5% chance that you will earn an A in both classes.

a. Find the probability that you do not get an A in either English or Physics.



$P(\text{not } E \cup \text{not physics})$

$= 1 - P(E \cup P)$

$= 1 - [0.2 + 0.1 - 0.05]$

$= \boxed{0.75}$

b. Are "earning an A in English" and "earning an A in Physics" disjoint events? Explain.

NOT disjoint b/c $P(E \cap P) \neq 0$

c. Are "earning an A in English" and "earning an A in Physics" independent events? Explain.

$P(E) \stackrel{?}{=} P(E \mid P)$

$P(E) \stackrel{?}{=} \frac{P(E \cap P)}{P(P)}$

$0.2 \stackrel{?}{=} \frac{0.05}{0.1}$

$0.2 \neq 0.5$

not independent

OR

$P(P) \stackrel{?}{=} P(P \mid E)$

$0.1 = \frac{P(P \cap E)}{P(E)}$

$0.1 = \frac{0.05}{0.2}$

$0.1 \neq 0.25$

not independent

OR

$P(P \cap E) \stackrel{?}{=} P(E) \cdot P(P)$

24. **Heights of adults** According to the National Health Survey, heights of adults may have a Normal model with mean heights of 69.1" for men and 64.0" for women. The respective standard deviations are 2.8" and 2.5."

a. Based on this information,

$\mu_M = 69.1$ $\quad \sigma_M = 2.8$

$\mu_W = 64.0$ $\quad \sigma_W = 2.5$

i) How much taller are men than women, on average?

$$E(M-W) = E(M) - E(W)$$
$$= 69.1'' - 64.0''$$
$$= \boxed{5.1''}$$

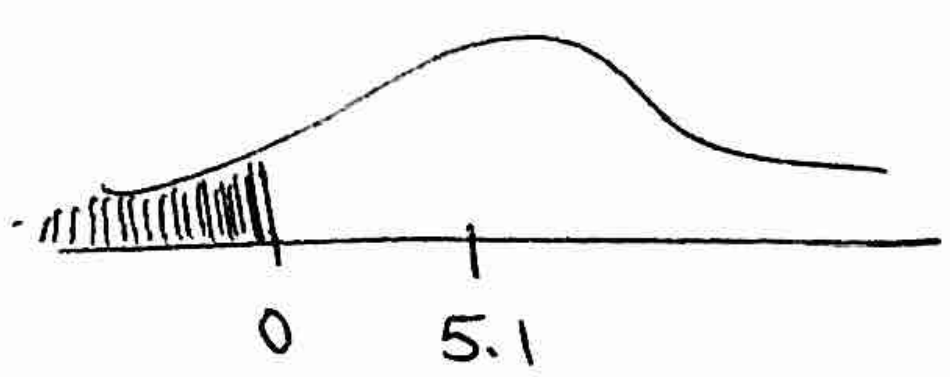
ii) What is the standard deviation for the difference in men's and women's heights?

$$\sigma_{M-W} = \sqrt{Var(M) + Var(W)} = \sqrt{2.8^2 + 2.5^2} = \boxed{3.75''}$$

b. Assume that women date men without considering the height of the man (i.e., that the heights of the couple are independent). What is the probability that a woman dates a man shorter than she is?

$N(5.1, 3.75)$

$D < 0$



$$z = \frac{0 - 5.1}{3.75} = -1.36 \qquad P(Z < -1.36) \approx \boxed{0.087}$$

OR

$$\text{normalcdf}(\underset{\text{lower}}{-10{,}000}, \underset{\text{upper}}{0}, \underset{\mu}{5.1}, \underset{\sigma}{3.75}) = 0.0869$$

25. **Home ownership** According to the Bureau of the Census, 68.0% of Americans owned their own homes in 2003. A local real estate office is curious as to whether a higher percentage of Americans own their own homes in its area. The office selects a random sample of 200 people in the area to estimate the percentage of those people that own their own homes.

a. Verify that a Normal model is a useful approximation for the Binomial in this situation.

$np = 200(0.68) = 136 \geq 10$

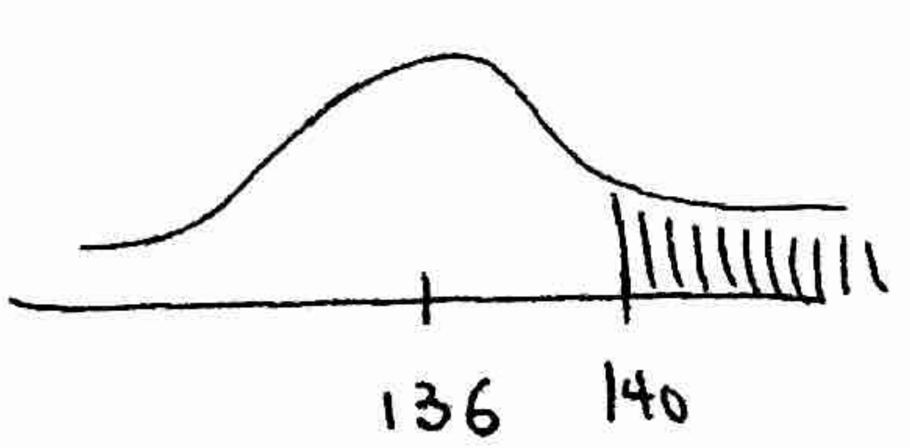
and $n(1-p) = 200(0.32) = 64 \geq 10$

and 200 people is less than 10% of the people in the area.

A normal model can be used.

b. What is the probability that at least 140 people will report owning their own home?

$$P(X \geq 140) = P(Z \geq 0.61) = \boxed{0.2709}$$



$$z = \frac{140 - 136}{6.597} = 0.61$$

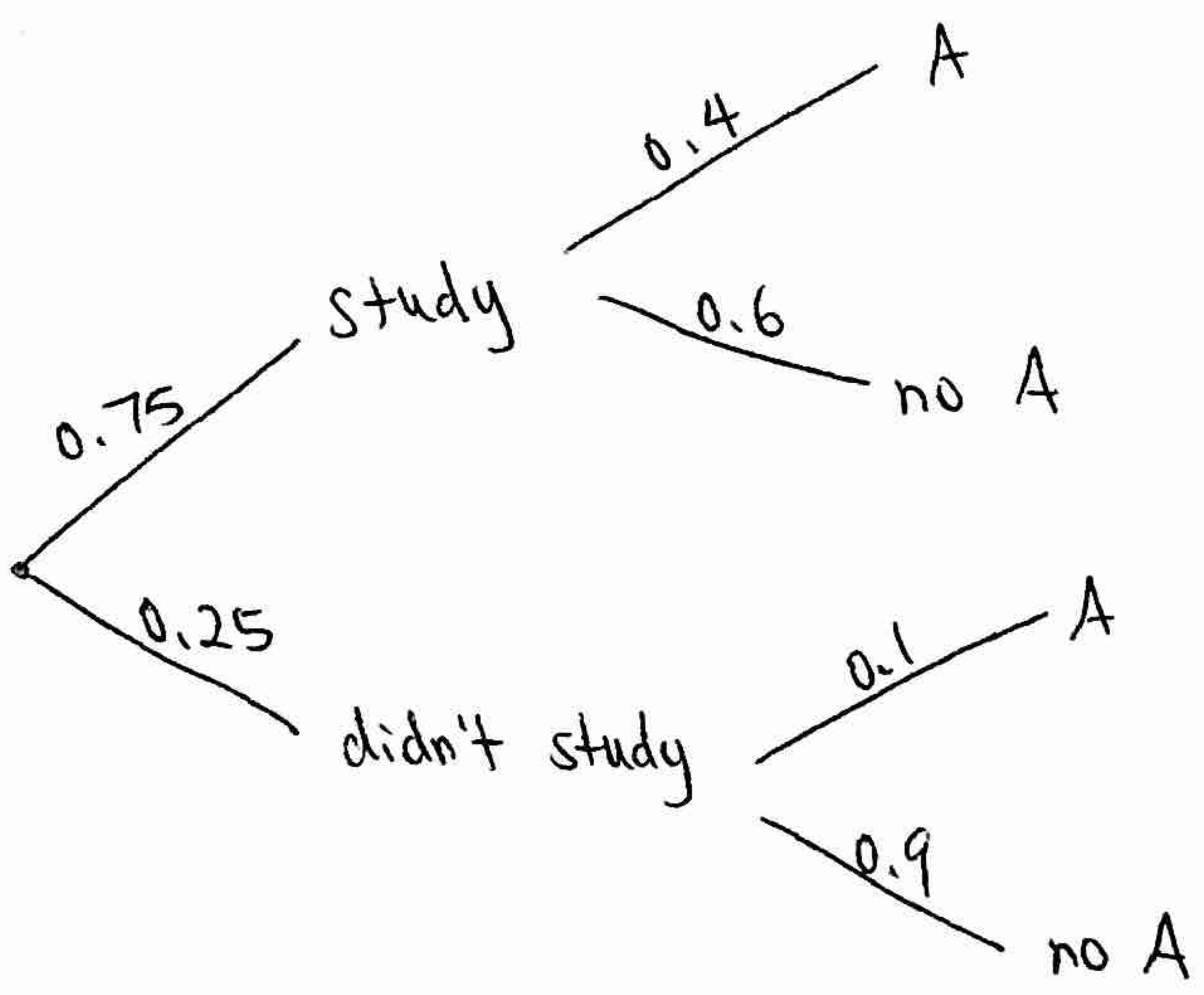
$\mu = np = 136$

$\sigma = \sqrt{np(1-p)} = \sqrt{200(0.68)(0.32)} = 6.597$

c. Based on the sample, how many people would it take for you to be convinced that a higher percentage of Americans own their own homes in that area? Explain.

It would be unusual to see a number of homeowners that was more than 2 s.d. above the mean. With a mean of 136 and s.d. of 6.60, it would be unusual to see $136 + 2(6.6) = 149.2$ or more homeowners in the area. I would be convinced that a higher percentage of Americans own their own homes in that area if at least 150 of the 200 people own their own homes.

26. **Studying** Assume that 75% of the AP Stat students studied for this test. If 40% of those who study get an A, but only 10% of those who don't study get an A, what is the probability that someone who gets an A actually studied for the test?



$$P(\text{studied} \mid A) = \frac{P(\text{studied} \cap A)}{P(A)}$$

$$= \frac{(0.75)(0.4)}{(0.75)(0.4) + (0.25)(0.1)}$$

$$= \boxed{0.923}$$