

**(WILL BE COUNTED AS 2 HW ASSIGNMENTS GRADE!)**

The following topics will appear on the Unit 5 Test:

- I. Sampling distributions
  - a. Sampling distribution of a sample proportion
  - b. Sampling distribution of a sample mean and CLT
- II. Estimation (point estimator and confidence interval)
  - a. Margins of error
  - b. Logic of confidence intervals and meaning of confidence level/confidence intervals
  - c. Large sample confidence interval for a proportion (i.e. 1-proportion z-interval)
  - d. Large sample confidence interval for a difference between two proportions (i.e. 2-proportion z-interval)
- III. Tests of significance
  - a. Logic of significance testing, null and alternative hypotheses, p-values, one- and two-sided tests,
  - b. Concepts of Type I and Type II errors, concept of power
  - c. Large sample test for a proportion
  - d. Large sample test for a difference between two proportions

The test will contain between 16-21 questions, a combination of multiple-choice and short answer. Additional tutoring will be available during tutoring hours in room 363 on Wednesday from 3:45-4:45PM.

**PART I:**

1. Suppose you are sampling from a distribution that is strongly skewed left. Which of the following statements about the sampling distribution of the sample mean is true?

- A) As the sample size increases, the shape of the sampling distribution gets closer and closer to a Normal distribution.
- B) As the sample size increases, the shape of the sampling distribution gets closer and closer to the shape of the population distribution.
- C) As the sample size increases, the mean of the sampling distribution gets closer to the population mean.
- D) Regardless of the sample size, the shape of the sampling distribution is similar to the shape of the population distribution.
- E) Regardless of the sample size, the standard deviation of the sampling distribution is approximately equal to the standard deviation of the population.

2. A relief fund is set up to collect donations for the families affected by recent storms. A random sample of 400 people shows that 28% of those 200 who were contacted by telephone actually made contributions compared to only 18% of the 200 who received first class mail requests. Which formula calculates the 95% confidence interval for the difference in the proportions of people who make donations if contacted by telephone or first class mail?

- A)  $(0.28 - 0.18) \pm 1.96 \sqrt{\frac{(0.23)(0.77)}{200}}$
- B)  $(0.28 - 0.18) \pm 1.96 \sqrt{\frac{(0.23)(0.77)}{200} + \frac{(0.23)(0.77)}{200}}$
- C)  $(0.28 - 0.18) \pm 1.96 \sqrt{\frac{(0.23)(0.77)}{400}}$
- D)  $(0.28 - 0.18) \pm 1.96 \sqrt{\frac{(0.28)(0.72)}{200} + \frac{(0.18)(0.82)}{200}}$
- E)  $(0.28 - 0.18) \pm 1.96 \sqrt{\frac{(0.28)(0.72)}{400} + \frac{(0.18)(0.82)}{400}}$

3. Which is true about a 95% confidence interval based on a given sample?

- I. The interval contains 95% of the population.
- II. Results from 95% of all samples will lie in the interval.
- III. The interval is narrower than a 98% confidence interval would be.

A) None                      B) I only                      C) II only                      D) III only                      E) II and III only

4. A truck company wants on-time delivery for 98% of the parts they order from a metal manufacturing plant. They have been ordering from Hudson Manufacturing but will switch to a new, cheaper manufacturer (Steel-R-U's) unless there is evidence that this new manufacturer cannot meet the 98% on-time goal. As a test the truck company purchases a random sample of metal parts from Steel-R-U's, and then determines if these parts were delivered on-time. Which hypothesis should they test?

- A)  $H_0 : p < 0.98$   
 $H_A : p > 0.98$       B)  $H_0 : p > 0.98$   
 $H_A : p = 0.98$       C)  $H_0 : p = 0.98$   
 $H_A : p < 0.98$       D)  $H_0 : p = 0.98$   
 $H_A : p \neq 0.98$       E)  $H_0 : p = 0.98$   
 $H_A : p > 0.98$

5. We are about to test a hypothesis using data from a well-designed study. Which is true?

- I. A small P-value would be strong evidence against the null hypothesis.
- II. We can set a higher standard of proof by choosing  $\alpha = 10\%$  instead of 5%.
- III. If we reduce the alpha level, we reduce the power of the test.

A) None                      B) I only                      C) II only                      D) III only                      E) I and III only

6. A pharmaceutical company investigating whether drug stores are less likely than food markets to remove over-the-counter drugs from the shelves when the drugs are past the expiration date found a P-value of 2.8%. This means that:

- A) 2.8% more drug stores remove over-the-counter drugs from the shelves when the drugs are past the expiration date.
- B) 97.2% more drug stores remove over-the-counter drugs from the shelves when the drugs are past the expiration date than drug stores.
- C) There is a 2.8% chance the drugstores remove more expired over-the-counter drugs.
- D) There is a 97.2% chance the drugstores remove more expired over-the-counter drugs.
- E) None of these.

7. To plan the course offerings for the next year a university department dean needs to estimate what impact the "No Child Left Behind" legislation might have on the teacher credentialing program. Historically, 40% of this university's pre-service teachers have qualified for paid internship positions each year. The Dean of Education looks at a random sample of internship applications to see what proportion indicate the applicant has achieved the content-mastery that is required for the internship. Based on these data he creates a 90% confidence interval of (33%, 41%). Could this confidence interval be used to test the hypothesis  $H_0 : p = 0.40$  versus  $H_A : p < 0.40$  at the  $\alpha = 0.05$  level of significance?

- A) Yes, since 40% is in the confidence interval he accepts the null hypothesis, concluding that the percentage of applicants qualified for paid internship positions will stay the same.
- B) Yes, since 40% is in the confidence interval he fails to reject the null hypothesis, concluding that there is not strong enough evidence of any change in the percent of qualified applicants.
- C) Yes, since 40% is not the center of the confidence interval he rejects the null hypothesis, concluding that the percentage of qualified applicants will decrease.
- D) No, because he should have used a 95% confidence interval.
- E) No, because the dean only reviewed a sample of the applicants instead of all of them.

8. Suppose that a conveyor used to sort packages by size does not work properly. We test the conveyor on several packages (with  $H_0$ : incorrect sort) and our data results in a P-value of 0.016. What probably happens as a result of our testing?

- A) We correctly fail to reject  $H_0$ .
- B) We correctly reject  $H_0$ .
- C) We reject  $H_0$ , making a Type I error.
- D) We reject  $H_0$ , making a Type II error.
- E) We fail to reject  $H_0$ , committing a Type II error.

9. We test the hypothesis that  $p = 35\%$  versus  $p < 35\%$ . We don't know it but actually  $p = 26\%$ . With which sample size and significance level will our test have the greatest power?

- A)  $\alpha = 0.01, n = 250$
- B)  $\alpha = 0.01, n = 400$
- C)  $\alpha = 0.03, n = 250$
- D)  $\alpha = 0.03, n = 400$
- E) The power will be the same as long as the true proportion  $p$  remains 26%

## PART II:

10. **Colorblind** Medical literature says that about 8% of males are colorblind. A university's introductory psychology course is taught in a large lecture hall. Among the students, there are 325 males. Each semester when the professor discusses visual perception, he shows the class a test for colorblindness. The percentage of males who are colorblind varies from semester to semester.

a) Is the sampling distribution model for the sample proportion likely to be Normal? Explain.

b) What are the mean and standard deviation of this sampling distribution model?

11. A certain beverage company is suspected of underfilling its cans of soft drink. The company advertises that its cans contain, on average, 12 ounces of soda with standard deviation 0.4 ounce. For the questions that follow, suppose that the company is telling the truth.

(a) A quality control inspector measures the contents of an SRS of 50 cans of the company's soda and calculates the sample mean  $\bar{x}$ . What are the mean and standard deviation of the sampling distribution of  $\bar{x}$  for samples of size  $n=50$ ?

(b) The inspector in part (a) obtains a sample mean of  $\bar{x} = 11.9$  ounces. Calculate the probability that a random sample of 50 cans produces a sample mean amount of 11.9 ounces or less. Be sure to explain why you can use a Normal calculation.

12. **Death penalty 2006.** In May of 2006, the Gallup Organization asked a random sample of 537 American adults this question:

*If you could choose between the following two approaches, which do you think is the better penalty for murder, the death penalty or life imprisonment, with absolutely no possibility of parole?*

Of those polled, 47% chose the death penalty, the lowest percentage in the 21 years that Gallup has asked this question.

a) Create a 95% confidence interval for the percentage of all American adults who favor the death penalty.

b) Based on your confidence interval, is it clear that the death penalty no longer has majority support? Explain.

c) If pollsters wanted to follow up on this poll with another survey that could determine the level of support for the death penalty to within 2% with 98% confidence, how many people should they poll?

13. **Eggs.** The ISA Babcock Company supplies poultry farmers with hens, advertising that a mature B300 Layer produces eggs with a mean weight of 60.7 grams. Suppose that egg weights follow a Normal model with standard deviation 3.1 grams.

a) What fraction of the eggs produced by these hens weigh more than 62 grams?

b) What's the probability that a dozen randomly selected eggs average more than 62 grams?

14. **Twins.** There is some indication in medical literature that doctors may have become more aggressive in inducing labor or doing preterm cesarean sections when a woman is carrying twins. Records at a large hospital show that, of the 43 sets of twins born in 1990, 20 were delivered before the 37th week of pregnancy. In 2000, 26 of 48 sets of twins were born preterm. Does this indicate an increase in the incidence of early births of twins? Test an appropriate hypothesis and state your conclusion.

15. **Pumpkin pie** A can of pumpkin pie mix contains a mean of 30 ounces and a standard deviation of 2 ounces. The contents of the cans are normally distributed. What is the probability that four randomly selected cans of pumpkin pie mix contain a total of more than 126 ounces?

16. **Approval rating** A newspaper article reported that a poll based on a sample of 1150 residents of a state showed that the state's Governor's job approval rating stood at 58%. They claimed a margin of error of  $\pm 3\%$ . What level of confidence were the pollsters using?

17. **Depression** A recent psychiatric study from the University of Southampton observed a higher incidence of depression among women whose birth weight was less than 6.6 pounds than in women whose birth weight was over 6.6 pounds. Based on a P-value of 0.0248 the researchers concluded there was evidence that low birth weights may be a risk factor for susceptibility to depression. Explain in context what the reported P-value means.

18. **Truckers** On many highways state police officers conduct inspections of driving logbooks from large trucks to see if the trucker has driven too many hours in a day. At one truck inspection station they issued citations to 49 of 348 truckers that they reviewed.

a. Based on the results of this inspection station, construct and interpret a 95% confidence interval for the proportion of truck drivers that have driven too many hours in a day.

b. Explain the meaning of “95% confidence” in part A.



## Quick Review

What do samples really tell us about the populations from which they are drawn? Are the results of an experiment meaningful, or are they just sampling error? Statistical inference based on our understanding of sampling models can help answer these questions. Here's a brief summary of the key concepts and skills:

→ Sampling models describe the variability of sample statistics using a remarkable result called the Central Limit Theorem.

- When the number of trials is sufficiently large, proportions found in different samples vary according to an approximately Normal model.
- When samples are sufficiently large, the means of different samples vary, with an approximately Normal model.
- The variability of sample statistics decreases as sample size increases.
- Statistical inference procedures are based on the Central Limit Theorem.
- No inference procedure is valid unless the underlying assumptions are true. Always check the conditions before proceeding.

→ A confidence interval uses a sample statistic (such as a proportion) to estimate a range of plausible values for the parameter of a population model.

- All confidence intervals involve an estimate of the parameter, a margin of error, and a level of confidence.
- For confidence intervals based on a given sample, the greater the margin of error, the higher the confidence.
- At a given level of confidence, the larger the sample, the smaller the margin of error.

→ A hypothesis test proposes a model for the population, then examines the observed statistics to see if that model is plausible.

- A null hypothesis suggests a parameter value for the population model. Usually, we assume there is nothing interesting, unusual, or different about the sample results.
- The alternative hypothesis states what we will believe if the sample results turn out to be inconsistent with our null model.
- We compare the difference between the statistic and the hypothesized value with the standard deviation of the statistic. It's the sampling distribution of this ratio that gives us a P-value.
- The P-value of the test is the conditional probability that the null model could produce results at least as extreme as those observed in the sample or the experiment just as a result of sampling error.
- A low P-value indicates evidence against the null model. If it is sufficiently low, we reject the null model.
- A high P-value indicates that the sample results are not inconsistent with the null model, so we cannot reject it. However, this does not prove the null model is true.
- Sometimes we will mistakenly reject the null hypothesis even though it's actually true—that's called a Type I error. If we fail to reject a false null hypothesis, we commit a Type II error.
- The power of a test measures its ability to detect a false null hypothesis.
- You can lower the risk of a Type I error by requiring a higher standard of proof (lower P-value) before rejecting the null hypothesis. But this will raise the risk of a Type II error and decrease the power of the test.
- The only way to increase the power of a test while decreasing the chance of committing either error is to design a study based on a larger sample.